M314 REVIEW EXERCISES 01.03.17

You're encouraged to discuss these problems with other students in the class.

1. Prove by induction that:
$$\forall n \in \mathbb{N},\ 3|(2^{2n}-1).$$

2. List all (integer) divisors of these numbers:
   - -12
   - 113
   - 100
   - 112

   Can you think of an efficient way to do this?
3. Find the greatest common divisor of these sets of numbers:
   - $\{-12, 112\}$
   - $\{113, 226\}$
   - $\{100, 24, 125\}$
   - $\{112, 252\}$
4. Which of these pairs of integers are coprime?
   - $\{113, 226\}$
   - $\{24, 125\}$
   - $\{-24, 226\}$
   - $\{17, 15\}$
   - $\{-1, 1\}$
5. Let m, n, a be integers. Then if $m|a,\ n|a$, and let $m,\ n$ be coprime. Prove that $m \cdot n|a$.
6. Define the complete set of integer solutions of these LDEs, the equations are given in the form $a \cdot x + b \cdot y = c$. You need to:
   - Check that solutions exist (i.e. that $GCD(a,b)|c$)
   - Express the $GCD(a,b)$ as a linear combination of a, b.
   - Multiply this expression by $\frac{c}{GCD(a,b)}$ to get one solution.
   - If x_0, y_0 is a solution, then so is:
$$x_n = x_0 + \frac{b}{GCD(a,b)}n,\ y_n = y_0 - \frac{a}{GCD(a,b)}n \quad \text{for any } n \in \mathbb{Z}$$

     Write an expression for the complete set.

   a) $97x + 35y = 13$

   b) $98x + 35y = 13$

   c) $258x + 147y = 369$