MTH314: Discrete Mathematics for Engineers

Lecture 9b: Introduction to Graph Theory

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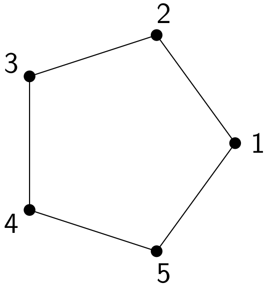
Graph Theory Basics

A graph $G = (V, E)$ is a data structure/mathematical object that consists of a set of vertices/nodes V and a relation E (edges) on this set.



V is the vertex set
$V = \{1, 2, 3, 4, 5\}$

E is the set of edges

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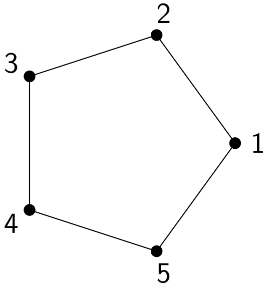
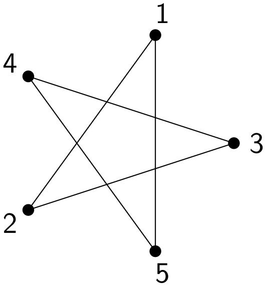




These two are the same graph. Just two different ways to draw it.

Graph: Definition

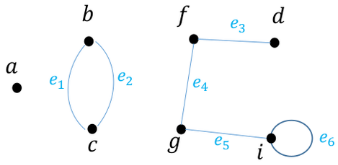
A graph $G = (V, E)$ is a data structure/mathematical object that consists of a set of vertices/nodes V and a relation E (edges) on this set.

Every edge $e \in E$ goes between two vertices, which we call *endpoints*. And edge from a vertex to itself is called a *loop*.

If there exists an $e \in E$ with endpoints $u, v \in V$ we say that u and v are adjacent, or that u is andjacent to v. We say that e is incident to both u and v.

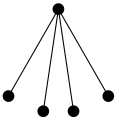
Example: $G = (V, E), V = \{a, b, c, d, f, g, i\},\ E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

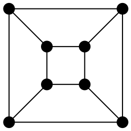
Edge	e_1	e_2	e_3	e_4	e_5	e_6
Endpoints	$\{b, c\}$	$\{b, c\}$	$\{f, d\}$	$\{f, g\}$	$\{g, i\}$	$\{i\}$



- e_6 is called a **loop**
- Vertex i is **adjacent** to itself
- Vertex a is **isolated**
- Edges e_1, e_2 are **parallel**
- Edges e_4, e_5 are **incident** to vertex g
- Vertices f, g are **connected** by edge e_4

The *degree* of a vertex is the number of edges coming out of that vertex. A loop will count twice, since both endpoints are at the same vertex.





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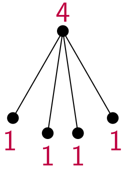
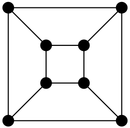
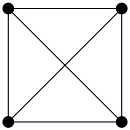




A graph $G = (V, E)$ is a *simple* if there are no edges from a vertex to itself ("loops") and between any two vertices there is at most one edge.

A *clique* is a simple graph where any two vertices are adjacent.



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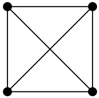
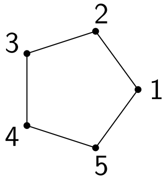
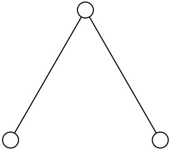


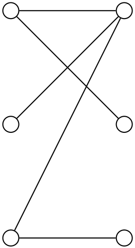
Not a clique:

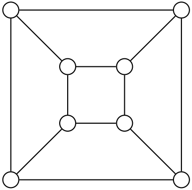


For example, vertices 1 and 3 are not adjacent.

A graph $G = (V, E)$ is called *bipartite* if there exists a partition of the set of vertices into two sets A and B, such that no two vertices in A are adjacent and no two verices in B are adjacent.







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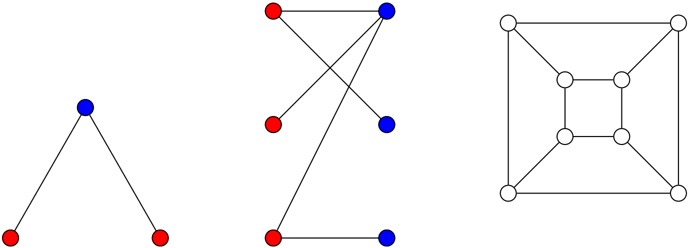

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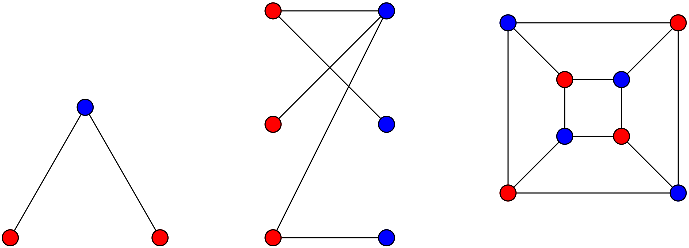
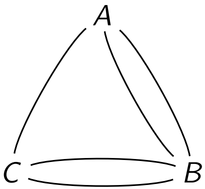


Adjacency Matrix

An adjacency matrix is an integer matrix that encodes the graph. Rows correspond to vertices, and columns correspond to vertices. i, j-entry (ith row and jth column) is the integer representing how many edges connect vertices i and j.



	A	B	C
A	0	2	1
B	2	0	2
C	1	2	0

In a simple graph, all entries are either 0 or 1 and all diagonal entries are 0. (why?)