RYERSON UNIVERSITY

Faculty of Science – Department of Mathematics

MTH 314 - Winter 2017

Assignment 1

Due date : Not Applicable

Guidelines:

- Assignment is optional, and will not be marked.
- Assignment solutions will not be posted.
- Assignment solutions can be discussed on piazza, or during office hours.
- Practising/solving these exercises is not only highly recommended, but also essential for doing well in the course.
- Assignment questions indicated by * are more challenging, and although they are recommended, you should not try them unless you are fully familiar with the non-star questions. Double-star questions, indicated by **, are even more difficult, and are meant to help you master the topic in depth. You should try any double-star questions only for leisure.
- You are highly encouraged to propose (detailed) solutions on piazza. Your answers **need to be fully justified**, unless specified otherwise. Always remember the WHAT-WHY-HOW rule, namely explain in full detail what you are doing, why are you doing it, and how are you doing it. Dry yes/no or numerical answers will receive no comments, and in exams they are worth 0 marks.

Question 1 What does it mean that AND is a *binary operation*? Explain why $(P \wedge Q) \wedge R$ makes sense syntactically, but $P \wedge Q \wedge R$ does not.

Question 2 Fill out the following truth table.

P	Q	$\neg Q$	$Q \wedge Q$	$Q \to P$	$P \to \neg P$	$\neg Q \vee P$	$Q \vee (Q \to P)$	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$
1	1							
1	0							
0	1							
0	0							

Which of the logical statements are logically equivalent to each other? Which represent tautologies? Which represent contradictions?
Hint: The last column is more complicated, because you may need some intermediate steps that are not provided by the previous columns. Write them out yourself.

Question 3 Fill each blank with either P or $\neg P$ to create a tautology. Use a truth table to make sure your answer is correct.

a) $P \vee$ ___

b) $(P \to Q) \vee$ ___

c) ___ $\to (Q \to P)$

d) $((P \to Q) \wedge$ ___ $) \to Q$

Question 4 Fill each blank with either P or $\neg P$ to create a contradiction. Use a truth table to make sure your answer is correct.

a) $P \wedge$ ___

b) $(P \to Q) \wedge ($ ___ $\wedge \neg Q)$

c) (___ $\wedge$ ___) $\to$ (___ $\vee$ ___)

Question 5 Use a truth table to prove the following logical equivalencies.

a) $P \vee Q \equiv Q \vee P$

b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

c) $P \to (Q \to R) \equiv Q \to (P \to R)$

For each of them, explain in plain English why they are intuitively correct.

Question 6 Use a truth table to determine whether the following are valid arguments. In each argument, the first line is a list of premises separated by commas, and the second line, beginning with $\therefore$ ("therefore"), is the conclusion.

a) $P \to Q,\ Q \to P$
 $\therefore\ P \vee Q$

b) $P,\ P \to Q,\ \neg Q \vee R$
 $\therefore\ R$

c) $P \vee Q,\ P \to \neg Q,\ P \to R$
 $\therefore\ R$

d) $P \wedge Q \to \neg R,\ P \vee \neg Q,\ \neg Q \to P$
 $\therefore\ \neg R$

Question 7 Find the contrapositive of the following statements. Verify that it's true.

a) $\neg(P \vee Q) \to \neg P$

b) $\neg Q \to P \wedge Q$

c) $\neg(P \wedge \neg Q) \to (P \to Q)$

Question 8 If we would like to prove the statement Q *by contradiction*, we show that $\neg Q$ is impossible.

Suppose that $(P \to Q) \wedge P$ is true. Prove by contradiction that Q is also true. In other words, assume $\neg Q$ and show that $(P \to Q) \wedge P$ must be false.

Which rows of the truth table are you crossing out now?

Question 9 Suppose that both $P \leftrightarrow Q$, $Q \to \neg(P \leftrightarrow Q)$ are true. Can you show by contradiction that both P and Q are false?

Question 10 * The binary operator "not and", denoted as $\uparrow$, is very important in engineering. $P \uparrow Q$ is formally defined as (or can be thought of as an abbreviation of) $\neg(P \wedge Q)$, hence its name. Notice that according to its definition, we have

P	Q	$P \uparrow Q$
1	1	0
1	0	1
0	1	1
0	0	1

A special thing about this operation is that you can create any other operation with multiple copies of NAND.

a) If you NAND P with itself, what unitary operation is that equivalent to? Fill out this truth table to figure it out:

P	$P \uparrow P$
1	
0	

b) How can you create $P \to Q$ out of P, Q using only the $\uparrow$ operator, and nothing else? You can use multiple copies of all of these building blocks. Write out a truth table to make sure you got it right. You may find it helpful to have extra columns in your truth table, one for each intermediate step.

c) Create $P \wedge Q$ and $P \vee Q$ out of P, Q and $\uparrow$. Test it with a truth table.

Question 11 ** According to the Question 6, we can create every binary operation just with multiple copies of NAND. For this reason, we say that NAND is *functionally complete*.
Can you come up with any other binary operation that is functionally complete?

Hint: Is OR functionally complete? Why or why not? Is there an operation we can't make with it?

Hint: To prove that another operator is also functionally complete, show that you can make NAND with it. Is this enough?
Fun fact: USB sticks are typically built entirely either with one type of the functionally complete logic gates or the other.