LECTURE 3 SUMMARY

1 Infinities

1.1 Hilbert's paradox of the Grand Hotel

We all have some notions of what *infinity* is. Let me tell you a story. Once upon a time, there was an infinite hotel. It had a room for every number and all rooms were occupied. And one day along came a new guest, and asked for a room. The receptionist was just about to tell him that they are fully booked, but the manager overheard it and suggested a solution: let's have every guest move to the NEXT room. So the guest in room 0 moved to room 1, the one in room 1 moved to room 2 and so on, and the new guest moved into room 0.

That story makes it clear that there are *countably* many rooms in the hotel, since they are numbered. You probably noticed that in the last definition of a probability distribution I used the word *countable*. If you've never come across it before, it means that there is a way to label all elements of a set with natural numbers and there won't be any left. Alternatively, there's a way to put all elements in one room of the infinite hotel each. A set like that has exactly as many elements at the set of natural numbers. That's the smallest infinity.

1.2 Example of sets that are countably infinite

1.3 Diagonalization argument

1.4 Infinite binary tree

2 Continuous Sample Spaces

2.1 Why density?

3 Monte Carlo Sampling

4 Counting the area