Lecture 11 **Math 20 Fall 2014, Dartmouth College**

1 Recap: funcions of random variables

For a strictly increasing function $Y = g(X)$, $Y \leq y$ if and only if $X \leq g^{-1}(y) = x$, i.e. the value x such that $g(x) = y$. So the cumulative distribution functions are related by:

$$F_Y(y) = F_X(g^{-1}(y)).$$

For a strictly decreasing function $Y = g(X)$:

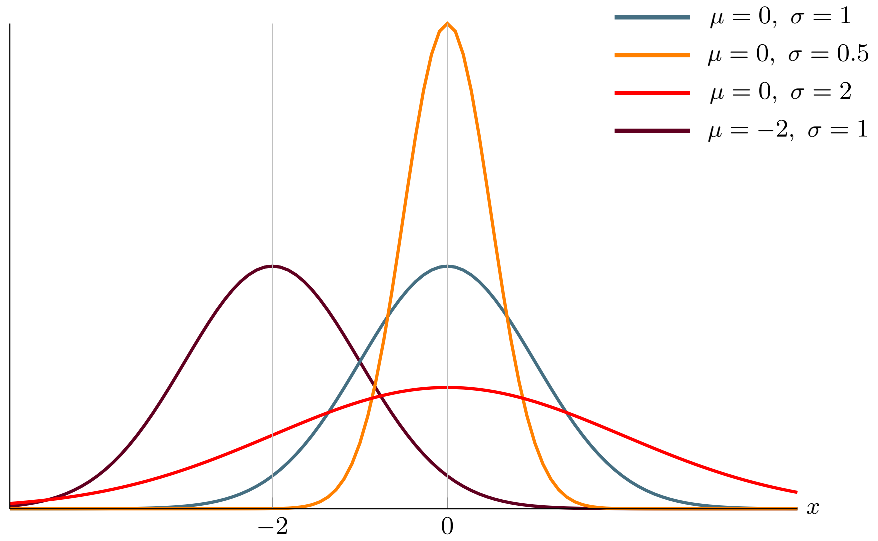
$$F_Y(y) = 1 - F_X(g^{-1}(x)).$$

2 Normal density

Normal density is by far the most important density function. Later in the course we will see the Central Limit Theorem, which says that when we sum a large number of independent random variables, the distribution of the sum will resemble the normal distribution. This is an incredible fact. It's also why most things in nature, such as height or weight of a population, are normally distributed. Because there are so many variables!

Normal density function takes two parameters, μ and σ with $\mu, \sigma \in \mathbb{R}$ and $\sigma > 0$. The first is the mean, the value of the variable where the density is centered. The second is the *standard deviation* and it measures how much the graph spreads out to the sides. The function is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$



This function is difficult to integrate, and it must be done numerically. Tables commonly exists for $\mu = 0,\ \sigma = 1$. Luckily, it's easy to derive any normal distribution function from that:

Let Z be a normal random variable with mean 0 and standard deviation 1. Let X be a normal random variable with mean μ and standard deviation σ. Then:

$$X = \sigma Z + \mu$$

and as we've seen,

$$F_X(x) = F_Z(\frac{x-\mu}{\sigma}).$$

Use a table to estimate the following values for for $\mu = 0,\ \sigma = 1$

$P(X \leq 0.2) =$

$P(0 \leq X \leq 1) =$

$P(0 \leq 2) =$

$P(0 \leq 0.75) =$

$P(X \geq 3) =$

Use a table for $\mu = 0,\ \sigma = 1$ to estimate the following values for $\mu = 75,\ \sigma = 15$:

$P(X \leq 75) =$

$P(55 \leq X \leq 90) =$

$P(X \leq 55) =$

$P(X \geq 90) =$

Example 1 On a test that determines whether an applicant receives a scholarship, the scores are distributed by a normal random variable with $\mu = 500,\ \sigma = 100$. It the top 5% of scores qualify for a scholarship, how high a score do you need to get it?

Let a be the value such that $P[X \geq a] = 0.05$. Then $P[X \leq a] = 0.95$.
For a standardized random variable Z, $P[Z \leq 1.65] \simeq 0.95$. So,

$$\frac{a-\mu}{\sigma} = 1.65$$

$$\frac{a-500}{100} = 1.65$$

$$a - 500 = 165$$

$$a = 665$$

3 Approximating a binomial random variable with normal distribution

A binomial random variable with parameters n, p can be approximated with a normal distribution with $\mu = np$ and $\sigma = np(1-p)$. We will see examples of that when we look at Central Limit Theorem.